Policy Impacts on Inequality

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by

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1.   SUMMARY

This module illustrates the simplest ways to measure inequality drawing on the statistical concepts of location, shape and variability. In particular, the following measures will be addressed: a) the range; b) the relative mean deviation; c) the variance; d) the coefficient of variation; and e) the standard deviation of logarithms. For all these measures, step-by-step procedures and numerical examples are also discussed.

2.   INTRODUCTION

Objectives

The objective of the tool is to focus on simple inequality measures and to discuss their relative merits as well as their relative disadvantages. Comparing and ranking income distributions is the natural outcome stemming from the use of inequality measurement.

Simple inequality measures are measures of inequality that are mainly rooted on statistical variability indexes. They are all mainly based on some measure of dispersion.

Measuring inequality is fundamental to understand the impact of policy changes on a given income distributions. In this way, alternative policies may be analysed and selected, based on their effect on income inequality.

Target audience

This module targets current or future policy analysts who want to increase their capacities in analysing impacts of development policies on inequality by means of income distribution analysis. On these grounds, economists and practitioners working in public administrations, in NGOs, professional organisations or consulting firms will find this helpful reference material.

Required background

Users should be familiar with basic notions of mathematics and statistics.

Links to relevant EASYPol modules, further readings and references are included both in the footnotes and in section 8.2 of this module[1].

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[1] EASYPol hyperlinks are shown in blue, as follows:
  a) training paths are shown in **underlined bold font**;
  b) other EASYPol modules or complementary EASYPol materials are in ***bold underlined italics***;
  c) links to the glossary are in **bold**; and
  d) external links are in *italics*.

3. CONCEPTUAL BACKGROUND

3.1. General issues

Statistics offer a variety of indexes to understand the characteristics of any income distribution. In order to develop a simple way to understand inequality, it is now worth recognizing what class of indexes are relevant for the analysis.

Statistics distinguishes indexes in three classes, Figure 1. There are three types of indexes:

      a) **location**

      b) **variability**

      c) **shape**.

The first type gives a synthesis of what is the representative value of the distribution. In this category, the main indexes are the **mode**, the **median** and the **mean**. The third type gives indications on what is the shape of the distribution (normal, asymmetric, peaked, etc.). **Skewness** and **kurtosis** are the main indexes to get information on shape. The second type is the kind of statistical indexes that inequality analysis has borrowed to have a synthesis of the attitude of the distribution to assume extreme values.

Figure 1: Statistical indexes

It is for this reason that the **range**, the **relative mean deviation**, the **variance** and the **coefficient of variation** have been long used to measure inequality in an income distribution. To these classical statistical measures, the tradition of inequality analysis adds the **standard deviation of logharitms**. It is to these measures that the next paragraphs are devoted.[2]

3.2. The range (R)

The range is the simplest measure of inequality. Given a vector representation of an income distribution $\mathbf{y} = (y_1, y_2,..., y_n)$ in a population with n individuals, the range is obtained by the ratio of the difference between the maximum and the minimum income and the mean income.

[1]
$$R = \left[\frac{Max(y_i) - Min(y_i)}{\bar{y}} \right]$$

In empirical applications, the range R is often replaced by a similar inequality index, the **interquantile difference**. In this latter case, the difference is not between the maximum and the minimum income, but between the cut points of two quantiles representing the two extremes of income distribution. One of the most used differences is between the cut points of the 95th and 5th percentile.

The range R has the following properties:

▪ **R has zero as minimum value**. When all incomes are equal, the numerator of R is equal to zero (Max=Min), so that $R=0$. **R has the number of observations n as maximum value**. When all incomes are zero but the last, the numerator is in fact total income. Total income divided by mean income gives $R=n$.[3] Therefore $0 \leq R \leq n$.

▪ **R is scale invariant**. By multiplying all incomes by a factor α, R remains unchanged, as both the numerator and the denominator increase by the same percentage α;

▪ **R is not translation invariant**. When all incomes of the original income distribution are added (subtracted) the same amount, the numerator of R remains unchanged, while the denominator increases (decreases) by that same amount. Therefore R decreases (increases).

▪ **R does not satisfy the Pigou-Dalton principle of transfers**, except in one special case, that of redistributing income between the two individuals at the extreme of the income distribution (the richest and the poorest). In this case, a redistribution from the richest to the poorest reduces R; the opposite occurs with a redistribution from

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[2] A good treatment of these measures can be found in Sen (1973) and Cowell (1977). Much of the material here presented is based on these contributions.

[3] In this case: $R = \dfrac{y\max - y\min}{\bar{y}} = \dfrac{Y - 0}{\dfrac{Y}{n}} = Y\dfrac{n}{Y} = n.$

the poorest to the richest. For any redistribution involving individuals who are not at the two extremes, R does not react to transfers.

In empirical application, the range R is often replaced by a similar inequality index, the **interquantile difference**. In this latter case, the difference is not between the maximum and the minimum income, but between the cut points of two quantiles representing the two extremes of the income distribution. One most used difference is between the cut points of the 95th and 5th percentile.

3.3. The relative mean deviation (RMD)

The relative mean deviation (RMD) compensates for the disadvantage of the range, by including the overall distribution in the measurement of inequality. Formally, RMD is calculated according to the following formula:

[2]
$$RMD = \frac{\sum_{i=1}^{n}|\bar{y} - y_i|}{Y}$$

It is easy to note that the numerator is the sum of the absolute values of the differences between mean income and individual incomes. The denominator is total income.

RMD has the following properties:

- **RMD has zero as minimum value**. When all incomes are equal, the numerator of RMD is equal to zero, as any difference between any income and the mean is zero.

  When all incomes are zero but the last, **RMD has maximum value at** $2\left[\frac{n-1}{n}\right]$.[4]

- **RMD is scale invariant**. By multiplying all incomes by a factor α, *RMD* remains unchanged, as both the numerator and the denominator increase by the same percentage α;

- **RMD is not translation invariant**. When all incomes of the original income distribution are added (subtracted) the same amount, the numerator of *RMD* remains unchanged, while the denominator increases (decreases) by an amount which is equal to the original addition (subtraction) times the number of observations. Therefore R decreases (increases).

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[4] In this particular case, the numerator is equal to: $RMD = \frac{(n-1)\bar{y} + (y_n - \bar{y})}{Y} = \frac{n\bar{y} - 2\bar{y} + y_n}{Y}$. By noting

that in this particular case $n\bar{y} = Y$ and also $y_n = Y$, it turns out that $RMD = 1 - \frac{2\bar{y}}{Y} + 1$. Finally, by noting

that $\frac{\bar{y}}{Y} = \frac{1}{n}$, it follows that $RMD = 2 - 2\left(\frac{1}{n}\right) = 2\left[\frac{n-1}{n}\right]$.

- **RMD satisfies the Pigou-Dalton principle of transfers.** When income is redistributed from richer to poorer, *RMD* decreases as the numerator decreases. The opposite holds true for redistributions from poorer to richer individuals. The effect of a given redistribution on *RMD* is the same whatever the relative position of individuals involved in the redistribution. It is however worth noting that RMD react to redistribution only for transfers across the mean. If the individuals involved in redistribution are all on the same side of the mean (either below or above), RMD does not change.

3.4. The variance (V)

The variance is the most common index of variability. It measures the dispersion of a distribution around the mean, but since differences from the mean are squared, it gives more weight to those values further away from the mean itself.

The formula for the variance is:

[3]
$$V = \frac{\sum_{i=1}^{n}(\bar{y} - y_i)^2}{n}$$

The numerator is now the sum of the squared differences of the distance between mean income and every income in the distribution (therefore all terms are positive); the numerator is total population.

The main properties of the variance are:

- When income is the same for all individuals, $V=0$ (the numerator of the formula is zero). This is the same as for **R** and **RMD**. However, the variance is unbounded from above.

- *V* depends on the measurement unit. For example, by multiplying all income of the baseline distribution by a number α, the variance increases by α^2. Therefore, it is not scale invariant. This is a very serious shortcoming when comparing different income distributions, for example, among countries.

- On the other hand, the variance is translation invariant.

- Transferring incomes among individuals changes the variance. In particular, it increases if the transfer occurs from a poorer to a richer person: it decreases if the transfer occurs from a richer to a poorer person. This seems an attractive property of inequality indexes. From this point of view, the variance performs better than *RMD*, which reacts only to those income transfers occurring across the mean.

To the purpose of measuring income inequality, it may be useful to consider not the variance by itself, but the ratio between the variance of the actual income distribution and the maximum variance that that same income distribution could generate were total

income concentrated in the hands of only one individual (a situation of maximum inequality).

[3b]
$$V(I) = \frac{V}{V\max}$$

By this transformation, the variance becomes a relative index, i.e. an index whose values range from zero to one. It will be zero if all incomes were equal (i.e. no variability). It will be one when all incomes are zero but one (as in this case $V=V\max$). It will be greater than zero and smaller than one in any other case.

The main properties of the index $V(I)$ are the following:

- **$V(I)$ ranges from 0 and 1**. It is zero when all incomes are equal, and in which case $V=0$. It is one when total income is concentrated in the hands of only one individual, and in that case $V=V\max$.

- **$V(I)$ is scale invariant**. By multiplying all incomes by the factor α, the variance of any distribution increases by α^2. In $V(I)$, therefore, both the numerator and the denominator increase by α^2 when all incomes of the original distribution are scaled by α. This means that the estimated $V(I)$ is the same.

- **$V(I)$ is not translation invariant**. When all incomes of the original income distribution are added (subtracted) the same amount, the variance V is in fact the same, but the variance $V\max$ increases (decreases). Therefore $V(I)$ decreases (increases).[5]

- **$V(I)$ satisfies the Pigou-Dalton principle of transfers**. Redistributing incomes from richer individuals to poorer individuals reduces measured inequality, as V decreases while $V\max$ remains unchanged, as total income in the society has not changed. Redistributing incomes from poorer to richer individuals increases measured inequality, as V increases while $V\max$ remains unchanged. The effect of a given transfer on $V(I)$, however, is greater when incomes involved are more distant from the mean.[6]

3.5. The coefficient of variation (CV)

The most troubling characteristic of the variance is its dependency from the measurement unit. As inequality analysis requires comparisons, it would be more useful to make the variance free from the measurement unit. The coefficient of variation does this job and is calculated as follows:

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[5] Note that this property is specific to the normalization chosen, i.e. to divide V by $V\max$. The variance V by itself is indeed translation invariant but not scale invariant, the opposite of $V(I)$.

[6] Deriving expression [3b] with respect to y would give: $\dfrac{\partial V(I)}{\partial y_i} = \dfrac{y_i - \bar{y}}{V\max}$.

[4]
$$CV = \frac{\sqrt{V}}{\bar{y}}$$

In expression [4], the numerator is the square root of the variance, i.e. the STANDARD DEVIATION. The denominator is the average income level.

CV has the following properties:

- When income is the same for all individuals, *CV*=0 (the numerator of the formula is zero because the variance is zero). This is the same as for *R*, *RMD* and V.

- It is scale invariant.

- It is not translation invariant.

- As in the case of *V*, *CV* satisfies the principle of transfers. The behaviour is the same as in *V*.

- The most important flaw of *CV* is that it is not bounded from above, i.e. it tends to become larger especially when mean income is low.

As in the case of variance *V*, it is useful to transform this index as follows:

[4b]
$$CV(I) = \frac{\sqrt{V} / \bar{y}}{\sqrt{V \max} / \bar{y}} = \frac{\sqrt{V}}{\sqrt{V \max}}$$

In this way, the relative coefficient of variation is simply the ratio between the standard deviation of the income distribution and the maximum standard deviation obtained by replacing all incomes but the last with zero incomes and by giving total income to the last individual.

CV(I) has the following properties:

- ***CV(I)* has zero as minimum value**. When all incomes are equal, *V*=0, so that the numerator of *CV(I)* is equal to zero. <u>*CV(I)* has one as maximum value</u>. When all incomes but the last are zero, *V*=Vmax.

- ***CV(I)* is scale invariant**. By multiplying all incomes by a factor α, the standard deviation of any distribution increases by α. In C*V(I)*, therefore, both the numerator and the denominator increase by α when all incomes of the original distribution are scaled by α. This means that the estimated *V(I)* is the same;

- ***CV(I)* is not translation invariant.** When all incomes of the original income distribution are added (subtracted) the same amount, the numerator of *CV(I)* is in

fact the same, but the denominator increases (decreases). Therefore $CV(I)$ decreases (increases);[7]

- **$CV(I)$ satisfies the Pigou-Dalton principle of transfers**. Redistributing incomes from richer to poorer individuals reduces measured inequality, as the standard deviation at the numerator decreases while the denominator remains unchanged. It is worth noting that $CV(I)$ decreases more if the redistribution from richer to poorer involves extremes.

3.6. The standard deviation of logharitms (SDL)

From the variance, we can obtain another inequality index, the standard deviation of logharitms. The starting point is the formula [3] for the variance. In this formula, the difference in the round brackets is replaced by the difference of logharitms of the same variable. Furthermore, a square root of the overall ratio is taken:

[5]
$$SDL = \sqrt{\frac{\sum_{i=1}^{n}\left(\ln \bar{y} - \ln y_i\right)^2}{n}}$$

SDL has the following main properties:

- When income is the same for all individuals, $SDL=0$ (the round bracket is zero). This is the same as for all other indexes.

- It is scale invariant.

- As inequality index, SDL has a peculiar type of sensitivity to income transfers. In particular, it attaches greater weight to income transfers occurring in the lower part of the income distribution. Intuitively, it means that when income transfers occur in the lower part of an income distribution, SDL decreases proportionally more than other inequality indexes.

4. A STEP-BY-STEP PROCEDURE TO BUILD SIMPLE INEQUALITY MEASURES

4.1. The range (R)

The step-by-step procedure needed to calculate simple inequality measures are usually easy to implement. figure 2 reports the procedure for the range R.

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[7] Note that this property is specific to the normalization chosen, i.e. to divide V by Vmax. The variance V by itself is indeed translation invariant but not scale invariant, the opposite of $V(I)$.

Figure 2: A step-by-step procedure to calculate the range

STEP	OPERATIONAL CONTENT
1	Sort the incombe distribution
2	Identify maximum and minimum incomes
3	Take the difference between the two
4	Calculate the average income level
5	Calculate the range R according to the formula [1] in the text

Step 1 asks us to sort the income distribution by income level. Step 2 asks us to identify the maximum and the minimum level of income in the income distribution.

In Step 3, we have to calculate the difference between these two extreme values. This is the numerator of the range. Step 4, instead, asks us to calculate the average income level. This is the denominator of the range.

Finally, Step 5 is the direct application of formula [1] in the text.

4.2. The relative mean deviation (RMD)

Figure 3 illustrates the procedure for the relative mean deviation (*RMD*). Step 1 asks us to sort the income distribution, while in Step 2 we have to calculate the average income level.

Figure 3: A step-by-step procedure to calculate the relative mean deviation

STEP	OPERATIONAL CONTENT
1	Sort the income distribution
2	Calculate the average income level
3	Take the difference between average income levels and each income
4	Calculate the absolute values of differences in Step 3 and take their sum
5	Calculate total income
6	Calculate RMD as in formula [2] in the text

In Step 3, have to take the difference between the average income level (Step 2) and each income level in the income distribution. These differences must first be taken in absolute values and then added together (Step 4).

Step 5 asks us to calculate the total income of the income distribution. After that, a direct application of formula [2] in the text gives the relative mean deviation.

4.3. The variance (V)

Figure 4 illustrates the procedure required to calculate the variance.

Step 1, as usual, asks us to sort the income distribution, while for Step 2 we have to calculate the average income level.

In Step 3, we must take the difference between the average income level and each income in the income distribution. These differences must be squared and added together (Step 4). Then, the population size has to be defined (Step 5).

Finally, the direct application of formula [3] gives the variance V (Step 6).

Figure 4: A step-by-step procedure to calculate the variance

STEP	OPERATIONAL CONTENT
1	Sort the income distribution
2	Calculate the average income level
3	Take the difference between the average income level and each income
4	Calculate the square of differences in Step 3 and take their sum
5	Define the population size n
6	Calculate V as in formula [3] in text

N.B. $V(I)$ can be simply obtained by dividing V by $Vmax$.

4.4. The coefficient of variation (CV)

Figure 5 illustrates the step-by-step procedure to calculate the coefficient of variation (CV). This procedure first asks us to calculate the variance as in the step-by-step procedure illustrated in Figure 4 (Step 1).

Step 2 asks us to calculate the square root of the variance V, while for Step 3 we have to calculate the average income level. Finally, Step 4 is the direct application of formula [4] in the text.

Figure 5: A step-by-step procedure to calculate the coefficient of variation

STEP	OPERATIONAL CONTENT
1	Calculate the variance V (see Figure 4)
2	Take the square root of the variance V
3	Calculate the average income level
4	Calculate CV as in formula [4] in the text

N.B.: *CV*(*I*) can be simply obtained by dividing *CV* by *CVmax*.

4.5.   The standard deviation of logarithms (SDL)

Figure 6 illustrates the step-by-step procedure to calculate the standard deviation of logarithms (SDL).

As usual, Step 1 asks us to sort the income distribution. Step 2, instead, wants us to calculate the average income level.

Figure 6: A step-by-step procedure to calculate the standard deviation of logarithms

STEP	OPERATIONAL CONTENT
1	Sort the income distribution
2	Calculate the average income level
3	Calculate the logarithm of the average income level
4	Calculate the logarithm of all income levels
5	Take the differences between the logarithm of average income levels and the average of each income
6	Take the square of the differences calculated in Step 5
7	Take the sum of the values calculated in Step 6
8	Define the population size n
9	Take the ratio between the values calculated in Step 7 and Step 8
10	Take the square root of the value calculated in Step 9

In Step 3, we must take the logarithm of the average income level, while in Step 4 each income level must also be transformed in logarithms. Step 5 asks us to take the difference between the log of the average income level and the log of each income. In Step 6, these differences must be squared, while in Step 7 we must take the sum of the values calculated in Step 6. This is the numerator of *SDL*.

Step 8 asks us to define the population size. This is the denominator of the *SDL*. The ratio between the numerator of Step 7 and the denominator of Step 8 is performed in Step 9. Finally, Step 10 asks us to take the square root of the value calculated in Step 9. This is *SDL*.

5. A NUMERICAL EXAMPLE TO CALCULATE SIMPLE INEQUALITY MEASURES

5.1. The range (R)

Table 1 illustrates the very simple procedure to calculate the range. First, sort the income distribution by income level. Then, identify maximum and minimum income levels. In the example below, the max level of income is 5,000 income units, while the minimum income level is 1,000 income units (Step 2). Afterwards, take the difference between these two levels. It gives 4,000 income units (Step 3). The average income level is 3,000 income units (Step 4). The range is simply (4,000/3,000), i.e. 1.333.

Table 1: A numerical example of how to calculate the range *R*

STEP 1	STEP 2	STEP 3	STEP 4	STEP 5
Sort the income distribution	Identify maximum and minimum levels of income	Take the difference between maximum and minimum level of income	Calculate the average income level	Calculate the range *R*

Individual	Incomes
1	1,000
2	2,000
3	3,000
4	4,000
5	5,000

Max income	5,000	Difference	4,000	Mean income	3,000	RANGE	1.333
Min income	1,000						

5.2. The relative mean deviation (RMD)

Table 2 reports a numerical example for the relative mean deviation. Step 1 always asks us to sort the income distribution by income level. In Step 2, the average income level is 3,000 income units.

Table 2: A numerical example of how to calculate the relative mean deviation *RMD*

STEP 1	STEP 2	STEP 3	STEP 4	STEP 5	STEP 6
Sort the income distribution	Calculate the average income level	Take the difference between the average income level and each income	Calculate absolute values of differences and take the sum	Calculate total income	Calculate *RMD*

Individual	Incomes		Individual	Differences	Individual	Absolute values of differences				
1	1,000		1	2,000	1	2,000				
2	2,000		2	1,000	2	1,000				
3	3,000		3	0	3	0				
4	4,000		4	-1,000	4	1,000				
5	5,000	Mean income 3,000	5	-2,000	5	2,000	Total income 15,000		**RMD**	0.400
						Sum 6,000				

Step 3 asks us to take the difference between 3,000 income units and each income level. This gives rise to the values reported in the column "Differences". To calculate RMD, we need to take the absolute values of these differences. This is done in Step 4, where the sum of these absolute values is also taken (6,000 income units).

Total income is then calculated in Step 5 (15,000 income units). The ratio between 6,000 and 15,000 income units yields the relative mean deviation as 0.400 (Step 6).

5.3. The variance (V)

Table 3 reports a numerical example to calculate the variance. The example can be extended to *V*(*I*) by dividing the result by *Vmax*. Step 1 again asks us to sort the income distribution by income level.

Table 3: A numerical example of how to calculate the variance *V*

STEP 1	STEP 2	STEP 3	STEP 4	STEP 5	STEP 6
Sort the income distribution	Calculate the average income level	Take the difference between the average income level and each income	Calculate the square of differences and take the sum	Define the population size	Calculate *V*

Individual	Incomes		Individual	Differences	Individual	Absolute values of differences				
1	1,000		1	2,000	1	4,000,000				
2	2,000		2	1,000	2	1,000,000				
3	3,000		3	0	3	0				
4	4,000		4	-1,000	4	1,000,000				
5	5,000	Mean income 3,000	5	-2,000	5	4,000,000	Pop. size 5		V	2,000,000
						Sum 10,000,000				

The average income level is calculated in Step 2 as 3,000 income units. Step 3 asks us to take the difference between the average income level and each income in the income distribution. This is done in Step 3. To calculate the variance, these differences must be squared. This is done in Step 4, where the sum of these squared differences is also taken (10,000,000).

Step 5 wants us to define the population size. In the example of Table 3 there are five individuals. Therefore, the population size is five. To calculate V, it is now sufficient to divide 10,000,000 by the population size. This gives 2,000,000, which is the variance.

5.4. The coefficient of variation (CV)

Table 4 illustrates how to calculate the coefficient of variation. Note that the example can be extended to $CV(I)$ by dividing the result by $Cvmax$. The starting point is the calculation of the variance V (Step 1). We are therefore referred to Figure 4 and Table 3 to work out the variance. Once this is done (2,000,000), Step 2 asks us to take the square root of the variance. This gives 1,414. Step 3 is the calculation of the average income level (3,000 income units). The coefficient of variation is the ratio between the square root of the variance and the average income level, i.e. 0.471 (Step 4).

Table 4 : A numerical example of how to calculate the coefficient of variation CV

STEP 1	STEP 2	STEP 3	STEP 4
Calculate V (*)	Take the square root of the variance	Calculate the average income level	Calculate CV
Total income — 2,000,000	Square root of V — 1,414	Mean income — 3,000	CV — 0.471

(*) see Table 3

5.5. The standard deviation of logarithms (SDL)

Table 5 reports the case of the standard deviation of logarithms. Step 1, as usual, asks us to sort the income distribution by income level. In Step 2, the average income level is calculated (3,000 income units).

Table 5: A numerical example of how to calculate the standard deviation of logarithms SDL

STEP 1		STEP 2	STEP 3	STEP 4		STEP 5		STEP 6 and 7		STEP 8	STEP 9	STEP 10
Sort the income distribution		Calculate the average income level	Calculate the logarithm of the average income level	Calculate the logarithm of the each income level		Take the differences between log of mean incomes and log of each income		Take the square of the differences		Define the population size	Take the ratio between the sum of squared differences and the population size	Calculate SDL
Individual	Incomes			Individual	Log(incomes)	Individual	Differences	Individual	Squared differences			
1	1,000			1	6.91	1	1.10	1	1.21			
2	2,000			2	7.60	2	0.41	2	0.16			
3	3,000			3	8.01	3	0.00	3	0.00			
4	4,000			4	8.29	4	-0.29	4	0.08	Pop. size 5	Ratio 0.343	SDL 0.586
5	5,000			5	8.52	5	-0.51	5	0.26			
		Mean income 3,000	Log (mean income) 8.01					Sum	1.72			

In Step 3 we have to calculate the logarithm of the average income level (8.01). In Step 4, the logarithms of each income level are also calculated. In Step 5, we must take the difference between the log of the average income level and the log of each income.

These differences must be squared (Step 6) and added together (Step 7). This gives rise to 1.72. Step 8 wants us to define the population size (five individuals). In Step 9, we take the ratio between 1.72 and 5, which is equal to 0.343. The standard deviation of logarithms is the square root of 0.343, i.e. 0.586.

6.   THE MAIN PROPERTIES OF SIMPLE INEQUALITY INDEXES

6.1. The range (R)

When using an inequality index, we would like to know how it performs under very basic changes of the income distribution. Table 6 illustrates the case for the range, but the framework will be the same for other simple inequality indexes.

Starting from a BASE income distribution, the following changes are analysed:

a)   an income distribution with equal incomes;

b)   an income distribution with all zero incomes except for the last, which owns total income;

c)   an income distribution obtained by the original one by increasing all incomes by 20 per cent;

d)   an income distribution obtained by the original one by increasing all incomes by 2,000 income units;

e)   an income distribution obtained by the original one by redistributing 500 income units from the richest to the poorest individual;

f)   an income distribution obtained by the original one by redistributing 500 income units from a rich to a poor individual (not the richest and the poorest).

Table 6: The main properties of R

BASE		Equidistribution		Max concentration		All income increased by 20 per cent		All income increased by 2,000 income units		Redistribution of 500 income units from the richest to the poorest		Redistribution of 500 income units from a rich to a poor	
Individual	Incomes	Individual	Incomes	Individual	Incomes	Individual	Incomes	Individual	Incomes	Individual	Incomes	Individual	Incomes
1	1,000	1	3,000	1	0	1	1,200	1	3,000	1	1,500	1	1,000
2	2,000	2	3,000	2	0	2	2,400	2	4,000	2	2,000	2	2,500
3	3,000	3	3,000	3	0	3	3,600	3	5,000	3	3,000	3	3,000
4	4,000	4	3,000	4	0	4	4,800	4	6,000	4	4,000	4	3,500
5	5,000	5	3,000	5	15,000	5	6,000	5	7,000	5	4,500	5	5,000
Range	1.333	Range	0	Range	5	Range	1.333	Range	0.800	Range	1.000	Range	1.333
			Minimum value		Maximum value, equal to the number of observations		UNCHANGED		DECREASED		DECREASED		UNCHANGED

The initial level of the range R is 1.333. When all incomes are equal the value of R is zero. When all incomes are zero but the last, R assumes a value equivalent to the number of individuals in the income distribution (5). Increasing all incomes by 20 per cent does not change R (1.333). Adding 2,000 income units to all incomes reduces R (0.800). Redistributing from the richest to the poorest reduces the value of R (1.000). Note, that **if redistribution does not involve extremes** (for the same mean income), **the value of R does not change**.

6.2. The relative mean deviation (RMD)

Table 7 illustrates the performance of the relative mean deviation.

Table 7: The main properties of the RMD

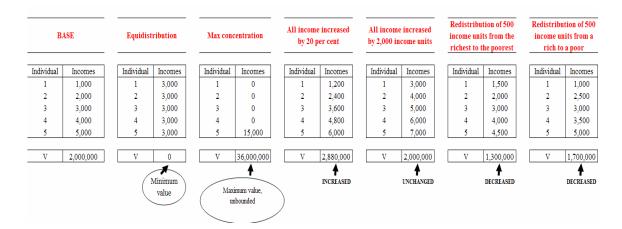| BASE		Equidistribution		Max concentration		All income increased by 20 per cent		All income increased by 2,000 income units		Redistribution of 500 income units from the richest to the poorest		Redistribution of 500 income units from a rich to a poor	
Individual	Incomes	Individual	Incomes	Individual	Incomes	Individual	Incomes	Individual	Incomes	Individual	Incomes	Individual	Incomes
1	1,000	1	3,000	1	0	1	1,200	1	3,000	1	1,500	1	1,000
2	2,000	2	3,000	2	0	2	2,400	2	4,000	2	2,000	2	2,500
3	3,000	3	3,000	3	0	3	3,600	3	5,000	3	3,000	3	3,000
4	4,000	4	3,000	4	0	4	4,800	4	6,000	4	4,000	4	3,500
5	5,000	5	3,000	5	15,000	5	6,000	5	7,000	5	4,500	5	5,000
RMD	0.400	RMD	0	RMD	1.600	RMD	0.400	RMD	0.240	RMD	0.333	RMD	0.333
			Minimum value		Maximum value, equal to $2[(n-1)/n]$		UNCHANGED		DECREASED		DECREASED		UNCHANGED

The initial level of RMD is 0.400. When all incomes are equal the value of RMD is zero. When all incomes are zero except for the last, RMD assumes the value 1.600, equivalent to $2[(n-1)/n]$. Increasing all incomes by 20 per cent does not change RMD.

Adding 2,000 income units to all incomes reduces RMD (0.240). Redistribution from the richest to the poorest reduces the value of RMD (0.333). If redistribution does not occur among extremes values (for the same mean income), the value of RMD is also reduces in the same way (0.333). It is however worth recalling that RMD reacts to redistribution only if transfers occur across the mean. **If transfers occur either below or above the mean, the value of RMD does not change**.

6.3. The variance (V)

Table 8 illustrates the performance of the variance.

Table 8: The main properties of V

BASE		Equidistribution		Max concentration		All income increased by 20 per cent		All income increased by 2,000 income units		Redistribution of 500 income units from the richest to the poorest		Redistribution of 500 income units from a rich to a poor	
Individual	Incomes	Individual	Incomes	Individual	Incomes	Individual	Incomes	Individual	Incomes	Individual	Incomes	Individual	Incomes
1	1,000	1	3,000	1	0	1	1,200	1	3,000	1	1,500	1	1,000
2	2,000	2	3,000	2	0	2	2,400	2	4,000	2	2,000	2	2,500
3	3,000	3	3,000	3	0	3	3,600	3	5,000	3	3,000	3	3,000
4	4,000	4	3,000	4	0	4	4,800	4	6,000	4	4,000	4	3,500
5	5,000	5	3,000	5	15,000	5	6,000	5	7,000	5	4,500	5	5,000
V	2,000,000	V	0	V	36,000,000	V	2,880,000	V	2,000,000	V	1,300,000	V	1,700,000
			Minimum value		Maximum value, unbounded		INCREASED		UNCHANGED		DECREASED		DECREASED

The initial level of V is 2,000,000. When all incomes are equal the value of *V* is zero. When all incomes are zero but the last, *V* assumes the value 36,000,000 which is unbounded, i.e. it depends on the size of incomes. Note that in this case, this is the value of *Vmax* needed for the calculation of *V(I)*. By increasing all incomes by 20 per cent increases V, i.e., it is not scale invariant. Adding 2,000 income units to all incomes, instead, leaves V unchanged. Redistributing from the richest to the poorest reduces the value of V (1,300,000), more than the case in which redistribution does not occur among extremes values (1,700,000).

6.4. The coefficient of variation (CV)

Table 9 illustrates the performance of the coefficient of variation.

Table 9: The main properties of CV

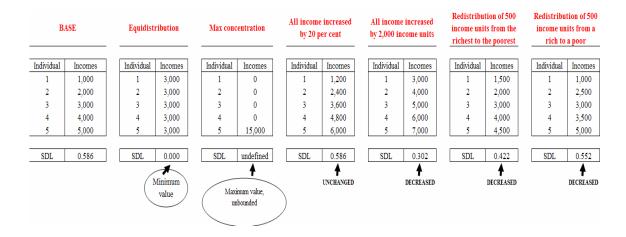| BASE		Equidistribution		Max concentration		All income increased by 20 per cent		All income increased by 2,000 income units		Redistribution of 500 income units from the richest to the poorest		Redistribution of 500 income units from a rich to a poor	
Individual	Incomes	Individual	Incomes	Individual	Incomes	Individual	Incomes	Individual	Incomes	Individual	Incomes	Individual	Incomes
1	1,000	1	3,000	1	0	1	1,200	1	3,000	1	1,500	1	1,000
2	2,000	2	3,000	2	0	2	2,400	2	4,000	2	2,000	2	2,500
3	3,000	3	3,000	3	0	3	3,600	3	5,000	3	3,000	3	3,000
4	4,000	4	3,000	4	0	4	4,800	4	6,000	4	4,000	4	3,500
5	5,000	5	3,000	5	15,000	5	6,000	5	7,000	5	4,500	5	5,000
CV	0.471	CV	0.000	CV	2.000	CV	0.471	CV	0.283	CV	0.380	CV	0.435
			Minimum value		Maximum value, unbounded		UNCHANGED		DECREASED		DECREASED		DECREASED

The initial level of *CV* is 0.471. When all incomes are equal the value of *CV* is zero. When all incomes are zero but the last, *CV* assumes the value of 2.000, which is not an upper bound. This value is the value of *CVmax* needed for the calculation of *CV(I)*. Increasing all incomes by 20 per cent does not change CV. Adding 2,000 income units to all incomes reduces CV (0.283). Redistributing from the richest to the poorest reduces the value of CV (0.380). If redistribution does not occur among extreme values (for the same mean income), the value of CV is also reduced, but less than in the previous case (0.435).

6.5. The standard deviation of logarithms (SDL)

Table 10 illustrates the performance of the standard deviation of logarithms.

The initial level of SDL is 0.586. When all incomes are equal the value of SDL is zero. When all incomes are zero except for the last, SDL is undefined, as the logarithm of zero cannot be calculated. Increasing all incomes by 20 per cent does not change SDL. Adding 2,000 income units to all incomes reduces SDL (0.302). Redistribution from the richest to the poorest reduces the value of SDL (0.422). If redistribution does not occur among extremes values (for the same mean income), the value of SDL is also reduced, but less than in the previous case (0.552).

Table 10: The main properties of SDL

BASE		Equidistribution		Max concentration		All income increased by 20 per cent		All income increased by 2,000 income units		Redistribution of 500 income units from the richest to the poorest		Redistribution of 500 income units from a rich to a poor	
Individual	Incomes	Individual	Incomes	Individual	Incomes	Individual	Incomes	Individual	Incomes	Individual	Incomes	Individual	Incomes
1	1,000	1	3,000	1	0	1	1,200	1	3,000	1	1,500	1	1,000
2	2,000	2	3,000	2	0	2	2,400	2	4,000	2	2,000	2	2,500
3	3,000	3	3,000	3	0	3	3,600	3	5,000	3	3,000	3	3,000
4	4,000	4	3,000	4	0	4	4,800	4	6,000	4	4,000	4	3,500
5	5,000	5	3,000	5	15,000	5	6,000	5	7,000	5	4,500	5	5,000
SDL	0.586	SDL	0.000	SDL	undefined	SDL	0.586	SDL	0.302	SDL	0.422	SDL	0.552
			Minimum value		Maximum value, unbounded		UNCHANGED		DECREASED		DECREASED		DECREASED

7. A SYNTHESIS

Table 11 offers a synthesis for the elements so far introduced in the treatment of simple inequality indexes. In particular, it gives a picture of the main characteristics of each index. For all indexes, the lower limit is zero (when incomes are all equal), while V, CV and SDL have unbounded upper limit. All of them, but the variance, are scale invariant. All indexes are sensitive to income transfers. However, V, CV and SDL are always transfer-sensitive, with SDL being more sensitive if these transfers occur in the lower part of the distribution. R and RMD are transfer-sensitive only in special cases.

Table 11: Features of simple inequality indexes

	LOWER LIMIT	UPPER LIMIT	SCALE INVARIANCE	TRANSLATION INVARIANCE	PRINCIPLE OF TRANSFERS	APPEAL
R	0	n	YES	NO	only if extremes involved	Low
RMD	0	$[2(n-1)]/n$	YES	NO	only if across the mean	Low
V	0	unbounded	NO	YES	YES	Medium
CV	0	unbounded	YES	NO	YES	High
SDL	0	unbounded	YES	NO	more weight to transfers occurring in the lower part of the distribution	High
V(I)	0	1	YES	NO	YES	High
CV(I)	0	1	YES	NO	YES	High

As a result, R and RMD have low appeal as inequality indexes; V has a medium degree of attraction, as it depends on measurement unit; CV and SDL, within this class of indexes, are certainly more appealing.

It is also worth noting the performance of V(I) and CV(I), ranging from zero to one, satisfying scale invariance and the principle of transfers but not scale invariance. Their appeal is high in this class of inequality indexes.

8.   READERS' NOTES

8.1.   Time requirements

Time required to deliver this module is estimated at about five hours

8.2.   EASYPol links

Selected EASYPol modules may be used to strengthen readers' background knowledge and to further expand their understanding on inequality and inequality measurement.

This module belongs to a set of modules which discuss how to compare on inequality grounds alternative income distributions generated by different policy options. It is part of the modules composing a training path addressing **Analysis and monitoring of socio-economic impacts of policies.**

The following EASYPol modules form a set of materials logically preceding the current module, which can be used to strengthen the user's background:

✓   EASYPol Module 000 : *Charting Income Inequality: The Lorenz Curve*

✓   EASYPol Module 001: *Social Welfare Analysis of Income Distributions: Ranking Income Distribution with Lorenz Curves*

✓   EASYPol Module 054: *Policy Impacts on Inequality: Inequality and Axioms for its Measurement*

8.3.   Frequently asked questions

▪   What is the simplest way of measuring inequality?

▪   Do simple inequality measures perform satisfactorily?

▪   What are the main shortcomings of simple inequality measures?

9.   REFERENCES AND FURTHER READING

Cowell F., 1977. *Measuring Inequality*, Phillip Allan, Oxford, UK.

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Pigou A.C., 1912. *Wealth and Welfare*, MacMillan, London, UK.

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Module metadata

1. EASYPol module	080

2. Title in original language	
English	Policy Impacts on Inequality
French	
Spanish	
Other language	

3. Subtitle in original language	
English	Simple Inequality Measures
French	
Spanish	
Other language	

4. Summary

This module illustrates the simplest ways to measure inequality drawing on the statistical concepts of location, shape and variability. In particular, the following measures will be addressed: a) the range; b) the relative mean deviation; c) the variance; d) the coefficient of variation; and e) the standard deviation of logharitms. For all these measures, step-by-step procedures and numerical examples are also discussed.

5. Date

December 2006

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7. Module type	☐ Thematic overview ☐ Conceptual and technical materials ☒ Analytical tools ☐ Applied materials ☐ Complementary resources
8. Topic covered by the module	☒ Agriculture in the macroeconomic context ☐ Agricultural and sub-sectoral policies ☐ Agro-industry and food chain policies ☐ Environment and sustainability ☐ Institutional and organizational development ☐ Investment planning and policies ☒ Poverty and food security ☐ Regional integration and international trade ☐ Rural Development
9.Subtopics covered by the module	
10. Training path	**Analysis and monitoring of socio-economic impacts of policies**
11. Keywords	capacity building, agriculture, agricultural policies, agricultural development, development policies, policy analysis, policy impact anlaysis, poverty, poor, food security, analytical tool, inequality, inequality axioms, inequality indexes, inequality measures, income inequality, income distribution, income ranking, welfare measures, social welfare, lorenz curve, lorenz dominance, theil index, gini index, relative mean deviation, variance, scale invariance,

translation principle of population, decomposability, simple inequality measures